# Numerical verification on Wavelet-based Vehicle Response Analysis method for bridge damage detection

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## ABSTRACT

VRA (Vehicle Response Analysis) is a method to evaluate the status of bridge structure only by using data measured on a traveling vehicle. The sensors are installed only on the vehicle. As analysis methods of VRA, Fourier's transform and other frequency domain techniques have been applied. However, they often showed lower accuracy than required for its commercial realization. On the other hand, several researchers have reported the high accuracy of time-frequency domain analysis for VRA, while those studies have verified the efficiencies of time-frequency domain analysis only by some simple numerical simulations and experiments. The aim of this study is to carry out a numerical simulation based on the 3D VBI (Vehicle-Bridge Interaction) system. Wavelet coefficients of acceleration response of a traveling vehicle are shown in this study and they are resulted that the high scales are efficient for damage detection.

#### BACKGROUND

The budget and human resources for bridge maintenance usually runs short. Thus, their reasonable allocation is needed. It means that the bridge screening method is in much demand, because it gives us the information about the urgency of each bridge. However, the bridge screening technology must be prompt and affordable. In this study, VRA (Vehicle Response Analysis) is focused on as a screening technology.

VRA is a method to evaluate the bridge status only by analyzing the vehicle vibration responses. Sensors are installed only on the passing vehicle, but not on the bridge. This brief inspection satisfies the both of conditions of time and cost in return of its lower accuracy. Yang et al. have proposed a VRA method and also shown that the peaks in the Fourier's PSD (Power Spectrum Densities) of the vehicle response accelerometers appear around the bridge's natural frequencies<sup>1</sup>. This approach, however, has two technical problems: the low estimation accuracy for the bridge natural frequencies, and the low sensitivity of the natural frequencies for the local damage of the bridge. The peaks of PSD in VRA just mean the predominant frequencies of the vehicle system. In fact, the predominant frequencies of the vehicle passing over the bridge are affected by the bridge system. However, the peaks are not always matching to the bridge natural frequencies exactly. Because of that, the estimation accuracy of the VRA based on the Fourier's transform tends to be low. On the other hand, the bridge damage we want to know usually appears in the limited range. This local damage cannot influence the global characteristics such as the frequency. This is the reason why the sensitivity of the natural frequencies to the bridge damage is often low.

To overcome these technical issues, time-frequency domain analysis is examined in this study. CWT (Continuous Wavelet Transformation) is one of the time-frequency domain analyses. CWT is developed to find the local changes in signals. The power of the signal can be obtained both in time and frequency domains. Thus, the application of CWT to VRA can be expected to be better suited for the local damage detection. Moreover, in the CWT analysis, "scale" is often used instead of "frequency". The predominant frequencies of the vehicle are not always same with the natural frequencies of the bridge. Thus, scale of CWT is used in this study. In the previous study<sup>2), 3)</sup>, the numerical simulation based on FE model shows that the local damage on the bridge girder causes a large change in the amplitude of wavelet coefficients calculated from the simulated vehicular accelerations. On the other hand, the authors carried out a field examination on an actual steel truss bridge. In the application of CWT to the data from that examination, however, it is very difficult to find the changes of wavelet coefficients before and after bridge damage.

Thus, in this study, the numerical model of bridge is updated to be able to simulate the dynamic behavior of truss bridge. The bridge is modeled by 3D FEM, while the vehicle is applied RBSM (Rigid-Body Spring Model) in the numerical simulation here. The damage is modeled by removing a member.

#### BASIS THEORY OF CWT

The equation of motion of the vehicle-bridge interaction (VBI) system which is modeled as continuous beam subject to n moving loads  $P_i(t)$  (i = 1, ..., n) can be described as Eq. (1).

$$\rho A \frac{\partial^2 y(x,t)}{\partial t^2} + C \frac{\partial y(x,t)}{\partial t} + \frac{\partial^2}{\partial x^2} \left( EI(x) \frac{\partial^2 y(x,t)}{\partial x^2} \right) = \sum_{i=1}^n P_i(t) \delta \left( x - x_i(t) \right)$$
(1)

where y(x, t) is bridge displacement at the location of x and time of t. And,  $\rho A$ , C and EI(x) are the mass per unit length, the damping and flexural stiffness of the bridge, respectively.  $P_i(t)$  is the *i*-th load and  $x_i(t)$  is its location at the time of t. n is the number of loads.  $\delta(x)$  is the Dirac delta function. The observation point in VRA travels over the bridge. Thus, its position can be written in  $x_m = vt$ . The observed bridge vibration is

$$f(t) = \frac{\partial^2}{\partial^2 t} (y(vt, t))$$
<sup>(2)</sup>

On the other hand, CWT of an integrable function f(t) is defined as

$$Wf(t,s) = f(t) \otimes \theta_s(t) = \int_{-\infty}^{\infty} f(t) \,\theta^* \left(\frac{t-u}{s}\right) du \tag{3}$$

where  $\theta_s(t)$  is so-called mother wavelet which is the dilation of window function  $\theta(t)$  by the scale *s*. *u* is the translation indicating the locality. ()\* denotes the complex conjugate. Wf(t,s) is called as the wavelet coefficient of f(t). Substitution of Eq. (2) into Eq. (3), we obtain

$$Wf(t,s) = f(t) \otimes \theta_{s}(t) = \frac{\partial^{2}}{\partial^{2}t} (y(vt,t)) \otimes \theta_{s}(x)$$
  
$$= y(vt,t) \otimes \frac{\partial^{2}\theta_{s}(t)}{\partial^{2}t}$$
(4)

If the Gaussian function is applied as  $\theta_s$ , the second order derivative of Gaussian function  $\partial^2 \theta_s(x_i)/\partial x_i^2$  is well known as the Mexican Hat wavelet that has the following explicit expression:

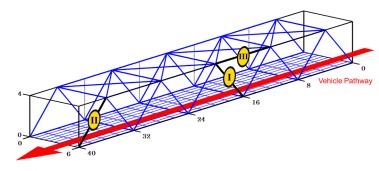
$$\frac{\partial^2 \theta(x)}{\partial x^2} = \frac{2}{\sqrt{3}} \pi^{-\frac{1}{4}} (x^2 - 1) e^{-x^2}$$
(5)

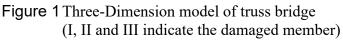
In this study, the wavelet coefficient calculated by Eq. (4) is proportional to the second derivative of y(vt, t) smoothed by the Gaussian window. Thus, CWT can be used to replace the bridge acceleration response with the displacement properties.

#### NUMERICAL SIMULATION OF VEHICLE-BRIDGE INTERACTION SYSTEM

In this study, the numerical simulation of VBI system is carried out. Figure 1 shows the 3D model of a truss bridge. The numbers (I, II and III) in this figure indicate the damage members corresponding to "Damage Case I", "Damage Case II" and "Damage Case III", respectively. The red arrow also means the vehicle pathway. All damage members considered in this study occurs in the side of the vehicle pathway. The damages are modeled by removing an objective member. Next, Figure 2 shows the vehicle model. RBSM is applied in this numerical simulation of VBI system. The properties of VBI system are shown in Table 1. The road profile is also considered and introduced to this simulation, based on the parameters of actual road roughness.

Table 2 shows the simulated acceleration vibrations and Fourier's PSDs of the rear unsprung-mass. As the result, the vibrations and their spectra of the damage cases are almost same with those of the intact case. The figures of the simulated responses in Table 2 are colored in blue for the intact case and in red for the damage





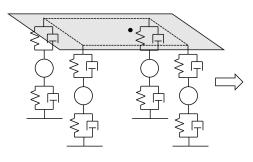


Figure 2 The RBSM (Rigid-Body Spring Model) of the passing vehicle

Table 1 The properties of VBI	(Vehicle Bridge Interaction) system
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(a) Vehicle Properties			(b) Bridge		
Sprung-	Mass [kg]	18,000	Global	Span Length [m]	40.0
	Damping [kg/s]	10,000		Width [m]	6.0
	Stiffness [kg/s <sup>2</sup> ]	$1.0 \times 10^{6}$	Deck	Element Axial Direction	20
	Inertia Moment (Pitch) [kg m <sup>2</sup> ]	65,000		Division Cross Direction	10
	Inertia Moment (Roll) [kg m <sup>2</sup> ]	15,000		Density [kg/m <sup>3</sup> ]	2400
	Length [m]	2.750		Thickness [m]	0.40
	Width [m]	1.800		Young's Modulus [Pa]	25×10 <sup>9</sup>
Unsprung-	Mass [kg]	1,100	Truss	Density [kg/m <sup>3</sup> ]	7800
	Damping [kg/s]	30,000 $3.5 \times 10^{6}$	Mem-	Young's Modulus [Pa]	200×10 <sup>9</sup>
	Stiffness [kg/s <sup>2</sup> ]		ber	Cross Section [m <sup>2</sup> ]	0.020
	Run speed [m/s]	10.0		Second Moment of Area [m <sup>4</sup> ]	1.0×10 <sup>-4</sup>
				Shear Modulus of Rigidity	78×10 <sup>9</sup>
				Second Polar Moment of Area [m <sup>4</sup> ]	1.0×10 <sup>-6</sup>

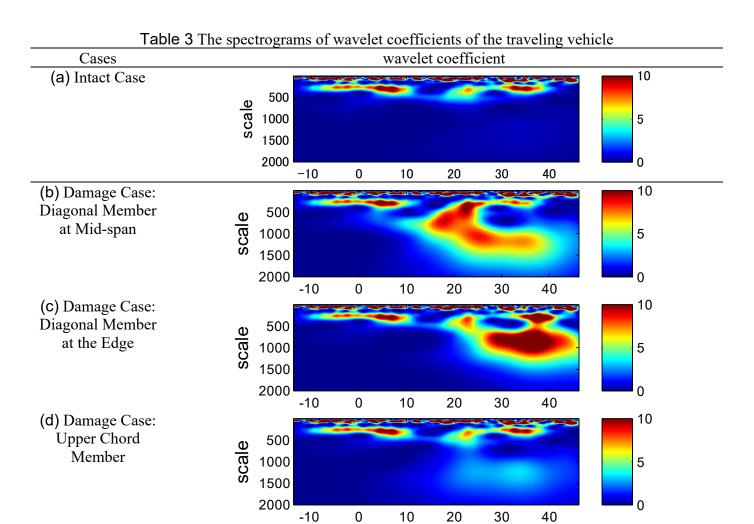
 Table 2 The acceleration response and its PSD of the traveling vehicle

Cases	Acceleration (m/s <sup>2</sup> )			Normalized Fourier's PSD					
(a) Intact Case	-2 0 -2 0 L/4	~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~	//////////////////////////////////////	-W//-\\	1 - 0 - 0	- <sup>12.0Hz</sup>	10.5H	z 	20
(b) Damage Case I:	2		Λο	ANN - N	1-	J	10.5H	z	-
Diagonal Member	-2	www.low.v/w/AM	1. Mrsh. Ann.	MAAAAA	0	12.0Hz	$\sim \sim $	hmm	
at Mid-span	0 L/4	2L/4	3L/4	L	0	5	10	15	20
(c) Damage Case II:	2				1	I	•10.2Hz		-
Diagonal Member	2 mmmm	wwwwwwww	hmhhh	-WV-v		2.0Hz	$\sim \sim $	hmm	
at the Edge	0 L/4	2L/4	3L/4	L	°0	5	10	15	20
(d) Damage Case III:	8 mmmmmm				1	· · · ·	•10.5H		_
Upper Chord	-3 WWWWWW	wwwwww	hmhhh	-WW-4		2.0Hz	$\sim$	hmm	
Member	0 L/4	2L/4	3L/4	L	0	5	10	15	20
	Normalized Vehicle Position				Frequency (Hz)				

ones. The horizontal axes of the figures of the acceleration vibration indicate the normalized vehicle position, but not time scale. L denotes the span length of the bridge. In figures in the right column, the horizontal axes indicate the frequency. The PSDs (power spectra distributions) are normalized to make each maximum value equal 1. Thus, the vertical axes are normalized power.

## **RESULT AND DISCUSSION**

The wavelet coefficients of vehicular acceleration responses are shown in the figures in Table 3. The considered four cases are (a) Intact Case in which all members are introduced into the bridge model, (b) Damage Case I, in which a diagonal member at the mid-span on the vehicle pathway side is removed, (c) Damage Case II, in which a diagonal member at the edge is removed, and (d) Damage Case III, in which the upper chord member over the mid-span is removed. The horizontal axes are the position of the rear unsprung-mass in these



figures. The range from x = 0 to L is the bridge section.

From the scale of 1 to about 500, it is difficult to find changes in the wavelet coefficients between intact and damage cases, while it is easy over the scale of 1000, after the rear unsprung-mass passes over the middle of the span. The high scale is corresponding to the low frequency components of the bridge dynamic responses. The higher wavelet coefficients over the scale of 1000 in the damage cases are caused by the effect of large deformation of the damaged bridge. This means the high feasibility of the application of CWT to VRA.

Vehicle Position [m]

## CONCLUSION

By the numerical simulation based on 3D FE model, the high scale of wavelet coefficients calculated from the vehicle vibration responses shows the differences after the bridge damage. Thus, the wavelet-based VRA has high feasibility. This conclusions matches the previous studies based on the simple simulations<sup>2), 3)</sup>.

## ACKNOWLEGEMENT

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## REFFERENCE

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